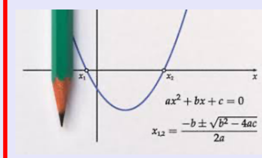
Math 125

Spring 2022

Lecture 22



Rationalizing the denominator:

"Remove the radicals from deno."

$$\frac{5}{\sqrt{10}} = \frac{5\cdot\sqrt{10}}{\sqrt{10}\,\sqrt{10}} = \frac{5\sqrt{10}}{\sqrt{100}} = \frac{\cancel{5}\sqrt{10}}{\cancel{10}_{2}} = \boxed{\frac{\sqrt{10}}{2}}$$

$$\frac{3x}{\sqrt{3x}} = \frac{3x\cdot\sqrt{3x}}{\sqrt{3x}\cdot\sqrt{3x}} = \frac{3x\sqrt{3x}}{\sqrt{9x^2}} = \frac{\cancel{3x}\sqrt{3x}}{\cancel{3x}} = \boxed{\sqrt{3x}}$$

Your turn: Rationalize the denominator

$$\frac{8x^2}{\sqrt{2x}} = \frac{8x^2\cdot\sqrt{2x}}{\sqrt{2x}\cdot\sqrt{2x}} = \frac{8x^2\sqrt{2x}}{\sqrt{4x^2}} = \frac{\overset{4\ x}{\cancel{8}\cancel{x^2}}\sqrt{2x}}{\cancel{2}\cancel{x}}$$

$$= \boxed{4x\sqrt{2x}}$$

When denominator has one of the following forms

$a+\sqrt{b}$, $a-\sqrt{b}$, $\sqrt{a}+b$, $\sqrt{a}-b$,

$\sqrt{a}+\sqrt{b}$, $\sqrt{a}-\sqrt{b}$

To rationalize the denominator, we multiply top and bottom by the conjugate of the denominator.

$$\frac{3}{\sqrt{5}-1}=\frac{3(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)}=\frac{3\sqrt{5}+3}{\underbrace{\sqrt{25}}_{5}\cancel{+\sqrt{5}}\cancel{-\sqrt{5}}-1}=\boxed{\frac{3\sqrt{5}+3}{4}}$$

$$\frac{\sqrt{5}}{\sqrt{5}+1}=\frac{\sqrt{5}(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)}=\frac{\sqrt{25}-\sqrt{5}}{\sqrt{25}\cancel{-\sqrt{5}}\cancel{+\sqrt{5}}-1}$$

$$=\frac{5-\sqrt{5}}{5-1}=\boxed{\frac{5-\sqrt{5}}{4}}$$

Rationalize the denominator

$$\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{6}(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$=\frac{\sqrt{18}+\sqrt{12}}{\sqrt{9}\cancel{+\sqrt{6}}\cancel{-\sqrt{6}}-\sqrt{4}}$$

$$=\frac{\sqrt{9}\sqrt{2}+\sqrt{4}\sqrt{3}}{3-2}=\frac{3\sqrt{2}+2\sqrt{3}}{1}=\boxed{3\sqrt{2}+2\sqrt{3}}$$

Your turn: Rationalize the denominator

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}=\frac{\sqrt{25}-\sqrt{15}-\sqrt{15}+\sqrt{9}}{\sqrt{25}\cancel{-\sqrt{15}}\cancel{+\sqrt{15}}-\sqrt{9}}$$

$$=\frac{5-2\sqrt{15}+3}{5-3}=\frac{8-2\sqrt{15}}{2}=\frac{\cancel{2}(4-\sqrt{15})}{\cancel{2}}$$

$$=\boxed{4-\sqrt{15}}$$

Rationalize the denominator:

$$\frac{2\sqrt{3}+\sqrt{2}}{\sqrt{6}-\sqrt{3}} = \frac{(2\sqrt{3}+\sqrt{2})(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})}$$

$$= \frac{2\sqrt{18}+2\sqrt{9}+\sqrt{12}+\sqrt{6}}{\sqrt{36}+\cancel{\sqrt{18}}-\cancel{\sqrt{18}}-\sqrt{9}}$$

$$= \frac{2\sqrt{9}\sqrt{2}+2\cdot 3+\sqrt{4}\sqrt{3}+\sqrt{6}}{6-3}$$

$$= \boxed{\frac{6\sqrt{2}+6+2\sqrt{3}+\sqrt{6}}{3}}$$

Solve & check

$$\sqrt{x}-\sqrt{x-3}=1$$

$$\sqrt{x}-1=\sqrt{x-3}$$

$$(\sqrt{x}-1)^2=(\sqrt{x-3})^2$$

$$(\sqrt{x}-1)(\sqrt{x}-1)=x-3$$

$$\sqrt{x^2}-\sqrt{x}-\sqrt{x}+1=x-3$$

$$\cancel{x}-2\sqrt{x}+1=\cancel{x}-3$$

$$-2\sqrt{x}=-3-1$$

$$-2\sqrt{x}=-4$$

$$\sqrt{x}=2$$

$$(\sqrt{x})^2=(2)^2$$

$$\boxed{x=4}$$

Check

$$\sqrt{4}-\sqrt{4-3}=1$$

$$\sqrt{4}-\sqrt{1}=1$$

$$2-1=1$$

$$1=1\checkmark$$

Solution Set $\rightarrow \{4\}$

Your turn:

$$\sqrt{x} + \sqrt{x+8} = 4$$

$$\sqrt{x+8} = 4 - \sqrt{x}$$

$$\left(\sqrt{x+8}\right)^2 = \left(4 - \sqrt{x}\right)^2$$

$$x+8 = (4-\sqrt{x})(4-\sqrt{x})$$

$$x+8 = 16 - 4\sqrt{x} - 4\sqrt{x} + \sqrt{x^2}$$

$$\cancel{x} + 8 = 16 - 8\sqrt{x} \cancel{+x}$$

$$8 - 16 = -8\sqrt{x}$$

$$-8 = -8\sqrt{x}$$

$$1 = \sqrt{x}$$

$$1^2 = \left(\sqrt{x}\right)^2$$

$$\boxed{x = 1}$$

Check

$$\sqrt{1} + \sqrt{1+8} = 4$$

$$1 + 3 = 4 \checkmark$$

$$\{1\}$$

Your turn:

$$\sqrt{2x+1} - \sqrt{x} = 1$$

$$\sqrt{2x+1} = 1 + \sqrt{x}$$

$$\left(\sqrt{2x+1}\right)^2 = \left(1 + \sqrt{x}\right)^2$$

$$2x+1 = (1+\sqrt{x})(1+\sqrt{x})$$

$$2x \cancel{+1} = \cancel{1} + \sqrt{x} + \sqrt{x} + \sqrt{x^2}$$

$$2x = 2\sqrt{x} + x$$

$$2x - x = 2\sqrt{x}$$

$$x = 2\sqrt{x}$$

$$(x)^2 = (2\sqrt{x})^2$$

$$x^2 = 4x$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$\checkmark \boxed{x=0} \quad \checkmark \boxed{x=4}$$

Check $x=0$ $\checkmark$

$$\sqrt{2(0)+1} - \sqrt{0} = 1$$

$$\sqrt{1} - 0 = 1 \checkmark$$

Check $x=4$

$$\sqrt{2(4)+1} - \sqrt{4} = 1$$

$$\sqrt{9} - \sqrt{4} = 1$$

$$3 - 2 = 1 \checkmark$$

$$\{0, 4\}$$

Solve & check

$\sqrt{3x+1} + \sqrt{x-4} = 5$

$\sqrt{3x+1} = 5 - \sqrt{x-4}$

$\left(\sqrt{3x+1}\right)^2 = \left(5 - \sqrt{x-4}\right)^2$

$3x + 1 = (5 - \sqrt{x-4})(5 - \sqrt{x-4})$

$3x+1 = 25 - 5\sqrt{x-4} - 5\sqrt{x-4} + \sqrt{(x-4)^2}$

$3x+1 = 25 - 10\sqrt{x-4} + x - 4$

$3x+1-25-x+4 = -10\sqrt{x-4}$

$2x - 20 = -10\sqrt{x-4}$

Divide by 2

$x - 10 = -5\sqrt{x-4}$

$(x-10)^2 = (-5\sqrt{x-4})^2$

$(x-10)(x-10) = 25(x-4)$

$x^2 - 10x - 10x + 100 = 25x - 100$

$x^2 - 20x + 100 - 25x + 100 = 0$

$x^2 - 45x + 200 = 0$

$(x-5)(x-40) = 0$

$x - 5 = 0 \qquad x - 40 = 0$

$\boxed{x=5} \qquad \boxed{x=40}$

Extraneous Solution

Check $\boxed{x=5}$ ✓

$\sqrt{3(5)+1} + \sqrt{5-4} = 5$

$\sqrt{16} + \sqrt{1} = 5$

$4 + 1 = 5$ ✓

Check $x=40$

$\sqrt{3(40)+1} + \sqrt{40-4} = 5$

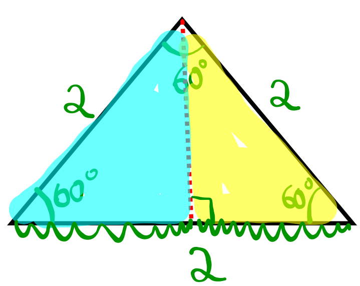
$\sqrt{121} + \sqrt{36} = 5$

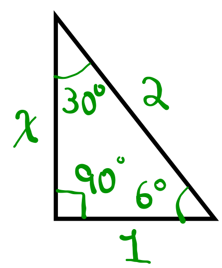
$11 + 6 = 5$

False

$\{5\}$

Consider the triangle below

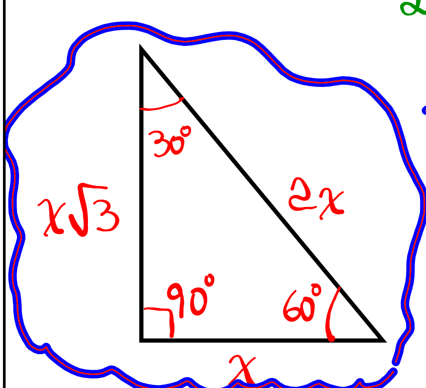




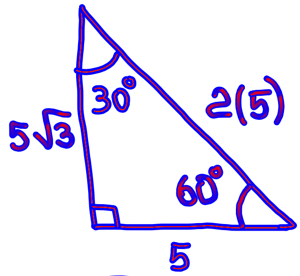
Using Pythagorean thrm

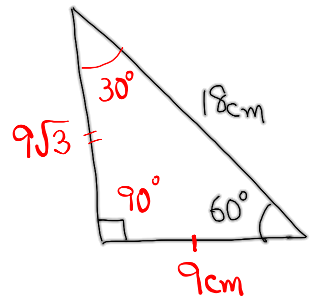
$x^2 + 1^2 = 2^2$

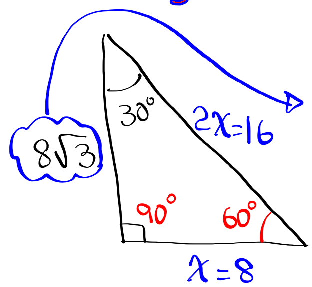
$x^2 = 3 \to x = \sqrt{3}$



30°-60°-90° right Triangle

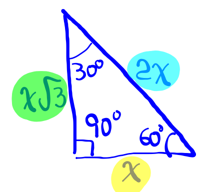




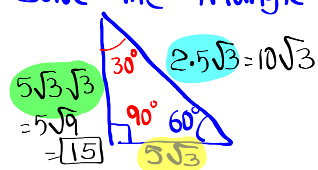


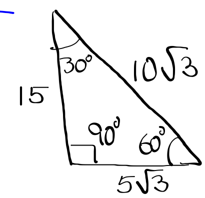
$x\sqrt{3}=8\sqrt{3}$

$x=8$

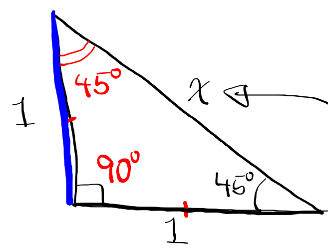


Solve the triangle below:





Consider the triangle below

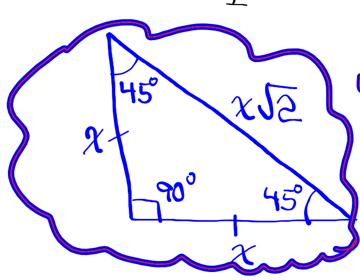


Using Pythagorean Thrm

$1^2+1^2=x^2$

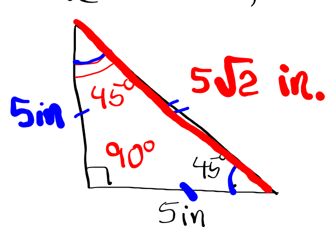
$2=x^2$

$\boxed{x=\sqrt{2}}$

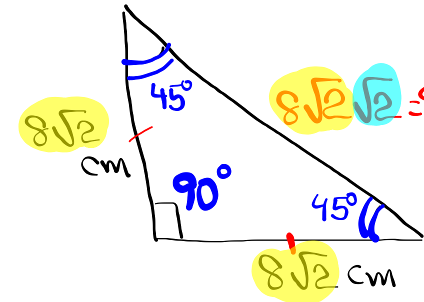


45°-45°-90° right-Triangle

Solve the triangle below

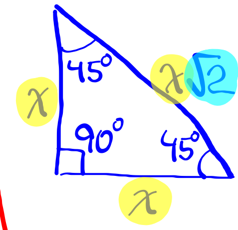


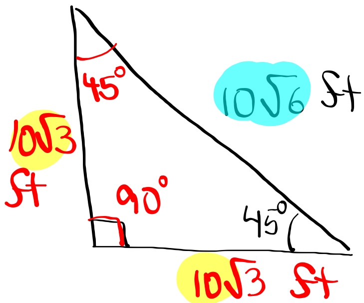
Solve the triangle below

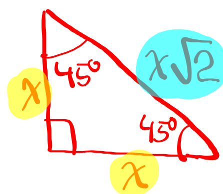


$8\sqrt{2}\sqrt{2} = 8\sqrt{4}$

$= 8 \cdot 2 = \boxed{16 \text{ cm}}$







$x\sqrt{2} = 10\sqrt{6}$

$x = \dfrac{10\sqrt{6}}{\sqrt{2}}$

$= \dfrac{10\sqrt{3}\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$x = 10\sqrt{3}$

Simplify

1) $(3-2i)^2 = (3-2i)(3-2i)$

$= 9 - 6i - 6i + 4i^2$

$= 9 - 12i + 4(-1) = \boxed{5-12i}$

2) $\dfrac{-25i}{3+4i} = \dfrac{-25i(3-4i)}{(3+4i)(3-4i)} = \dfrac{-75i + 100i^2}{9 \cancel{-12i} \cancel{+12i} - 16i^2}$

$= \dfrac{-75i + 100(-1)}{9 - 16(-1)} = \dfrac{-100 - 75i}{25}$

$= \dfrac{-100}{25} - \dfrac{75}{25}i$

$= \boxed{-4-3i}$

Simplify

1) $\sqrt{-64} = \sqrt{64}\sqrt{-1} = \boxed{8i}$

2) $-3\sqrt{-25} = -3\sqrt{25}\sqrt{-1} = -3\cdot 5i = \boxed{-15i}$

3) $\sqrt{-4}\,\sqrt{-100} = 2i \cdot 10i = 20i^2 = 20(-1)$

$= \boxed{-20}$

4) $\sqrt{-98} - \sqrt{-50}$

$= \sqrt{49}\sqrt{2}\sqrt{-1} - \sqrt{25}\sqrt{2}\sqrt{-1}$

$= 7\sqrt{2}\,i - 5\sqrt{2}\,i = \boxed{2\sqrt{2}\,i}$ ✓

$\cancel{2\sqrt{2i}}$ WRONG $= \boxed{2i\sqrt{2}}$ ✓

Simplify (Assume all variables are non-negative)

$\sqrt{45x^3} = \sqrt{9x^2}\sqrt{5x}$

$= \boxed{3x\sqrt{5x}}$

$\sqrt[3]{40x^4y^8} = \sqrt[3]{8x^3y^6}\,\sqrt[3]{5xy^2}$

2^3

$= \sqrt[3]{2^3x^3y^3y^3}\,\sqrt[3]{5xy^2}$

$= 2xyy\,\sqrt[3]{5xy^2}$

$= \boxed{2xy^2\sqrt[3]{5xy^2}}$

Simplify

$$\sqrt[4]{32x^5y^6z^7} = \sqrt[4]{\cancel{2^4}\cancel{x^4}\cancel{y^4}\cancel{z^4}}\sqrt[4]{2xy^2z^3}$$

$32 = 2^5$

$$= \boxed{2xyz\sqrt[4]{2xy^2z^3}}$$

Class QZ 16

1) Simplify: $(3-2i)(4+3i)$

$= 12 + 9i - 8i - 6i^2$

$= 12 + i - 6(-1) = \boxed{18+i}$

2) Divide: $\dfrac{5i}{1+2i} = \dfrac{5i(1-2i)}{(1+2i)(1-2i)}$

$= \dfrac{5i - 10i^2}{1 - 2i + 2i - 4i^2} = \dfrac{5i - 10(-1)}{1 - 4(-1)}$

$= \dfrac{5i + 10}{1+4} = \dfrac{10}{5} + \dfrac{5}{5}i = \boxed{2+i}$